Report 3675

STREAMLINE GEOMETRY AND EQUIVALENT RADIUS OVER A FLAT DELTA WING WITH CYLINDRICAL LEADING EDGE AT ANGLES OF ATTACK

NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

Bethesda, Maryland 20034

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by

Tsze C. Tai

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DEPARTMENT OF THE NAVY
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NOTATION

b	Span of a wing
C_1, C_2	Functional constants in Equation (32)
E	Viscous correction in Equation (33)
h_1, h_2	Scale factors for curvilinear coordinates ξ and β, respectively; h_2 is also called equivalent radius
$\bar{h}_2$	Equal to h_2/R_0
L	A length segment
M	Mach number
P	Static pressure
R_0	Nose radius
r	Body radius measured from body axis
S	Distance along a streamline measured from the stagnation point
$\bar{S}$	Equal to S/R_0
u,v	Inviscid velocity components in (x,ϕ) coordinates
u_e	Velocity at the edge of boundary layer
V_∞	Free-stream velocity
V_1, V_2, V_3	Velocity components in curvilinear coordinates ξ, β, ζ
x, ϕ, z	Body-oriented orthogonal coordinates
$\bar{x}$	Equal to x/R_0
α	Angle of attack
$\bar{\gamma}$	Effective ratio of specific heats after shock
θ	Angle between the tangent to a local streamline and the radial line
ξ, β, ζ	Streamline-oriented, orthogonal, curvilinear coordinates
ρ	Density

<u>Subscripts</u>

e	At the edge of boundary layer
o	Stagnation conditions at the edge of boundary layer
∞	Undisturbed, free-stream condition

ABSTRACT

An exact method has been developed for determining the streamline geometry and equivalent radius (the scale factor for the normal coordinate in a streamwise coordinate system) over a flat delta wing with cylindrical leading edge at angles of attack. This method requires a knowledge of the surface inviscid pressure distribution, either theoretical or experimental. With the aid of the present method, three-dimensional hypersonic heat transfer can be simply calculated through the axisymmetric analogue.

Results are presented for flat delta wings traveling at hypersonic speeds and at various angles of attack. Calculated results indicate that the streamline geometry depends heavily on pressure distribution used in the calculation. The present method gives better correlation with experimental data than does the perturbation method.

ADMINISTRATIVE INFORMATION

The work presented in this report was supported jointly by the Naval Air Systems Command and NASA Marshall Space Flight Center under NAVAIR TASK A370538B and NASA Purchase Request H58564A.

INTRODUCTION

The calculation of three-dimensional viscous flows can be substantially simplified by using streamwise coordinates. This has become a popular procedure for computing hypersonic heat transfer to bodies at an angle of attack.[1-6*] The most obvious difficulty is how to determine the three-dimensional inviscid streamline pattern and its inherent equivalent radius.

The available literature contains a few analyses for obtaining the inviscid streamline geometry.[4,7-9] Previous work by the present author[9] provides the most straightforward method. It gives exact equations for determining the surface streamline pattern and equivalent radius over bodies of revolution at an angle of attack. The method requires knowledge of the pressure distribution, either theoretical or experimental. It has been successfully applied to cases covering hypersonic laminar and turbulent heat transfer to bodies of revolution at particular angles of attack.

In an extension of previous work, this report describes a method developed for determining the streamline geometry and equivalent radius over a flat delta wing with cylindrical leading edge at an angle of attack. Results are presented for the cases of a flat delta wing and a cylindrical leading edge at hypersonic speeds.

ANALYSIS

COORDINATE SYSTEMS

Two coordinate systems were used:

1. The body-oriented orthogonal system with coordinates x, ϕ, and z. For this system,

$$dL^2 = dx^2 + (x d\phi)^2 + dz^2$$

2. The streamwise orthogonal system with coordinates ξ, β, and ζ. For this system,

$$dL^2 = (h_1 d\xi)^2 + (h_2 d\beta)^2 + d\zeta^2$$

They are shown in Figures 1 and 2.

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*References are listed on page 21.

The body-oriented orthogonal coordinates (x,ϕ,z) are employed as a reference frame upon which the three-dimensional inviscid flow equations are written. (Note that the z-coordinate is always zero at body surface.) The streamwise surface coordinates (ξ,β,ζ) are calculated in terms of body coordinates x and ϕ. The scale factor h_2 for the β coordinate must also be calculated along with the streamline geometry. Since the scale factor h_2 appears in the three-dimensional flow, equations written in streamwise coordinates correspond to the radius for an axisymmetric system; it is referred to here as "equivalent radius" to give more physical meaning.

EQUATIONS FOR STREAMLINE GEOMETRY

In body-oriented orthogonal coordinates, the inviscid momentum equations along the surface of a general three-dimensional body are:

x-momentum

$$u\frac{\partial u}{\partial x} + \frac{v}{x}\frac{\partial u}{\partial \phi} - \frac{v^2}{x} = -\frac{1}{\rho}\frac{\partial P}{\partial x} \tag{1}$$

ϕ-momentum

$$u\frac{\partial v}{\partial x} + \frac{v}{x}\frac{\partial v}{\partial \phi} + \frac{uv}{x} = -\frac{1}{\rho x}\frac{\partial P}{\partial \phi} \tag{2}$$

where x is the distance along the body surface of a constant ϕ plane and ϕ is the azimuthal angle measured from a reference position. It is convenient to measure ϕ from the leading edge if the streamline pattern is a converging flow or from the centerline if it is a diverging flow (see Figure 1). The velocity components u and v are measured along the surface in the x and ϕ directions, respectively, and P is the static pressure.

The geometry of any streamline emanating from the stagnation point may be expressed as $\phi = \phi(x,\beta)$; here β is constant along a streamline. The coordinates are related to the velocity components through the relation

$$\frac{x\,d\phi}{dx} = \frac{v}{u} \tag{3}$$

Defining D/Dx as the substantial derivative, or derivative along a streamline, Equation (3) can be written in the form

$$\frac{D\phi}{Dx} = \frac{v}{xu} \tag{4}$$

Differentiating the above equation with respect to x yields

$$\frac{D^2\phi}{Dx^2} = \frac{1}{x}\left[\frac{u\frac{Dv}{Dx} - v\frac{Du}{Dx}}{u^2} - \frac{v}{xu}\right] \tag{5}$$

Since $u = u(x,\phi)$, then

$$Du = \frac{\partial u}{\partial x} Dx + \frac{\partial u}{\partial \phi} D\phi$$

and using Equation (4), the above expression can be written as

$$\frac{Du}{Dx} = \frac{\partial u}{\partial x} + \frac{v}{xu}\frac{\partial u}{\partial \phi} \tag{6}$$

Using this result in Equation (1), one gets

$$\frac{Du}{Dx} = \frac{1}{u}\left(\frac{v^2}{x} - \frac{1}{\rho}\frac{\partial P}{\partial x}\right) \tag{7}$$

In a similar manner, Equation (2) gives

$$\frac{Dv}{Dx} = -\frac{1}{x}\left(uv + \frac{1}{\rho}\frac{\partial P}{\partial \phi}\right) \tag{8}$$

Since $V^2 = \bar{\gamma}M^2P/\rho$, it follows that

$$u^2 = \frac{\bar{\gamma}M^2P}{\rho} \frac{1}{1 + x^2\left(\frac{D\phi}{Dx}\right)^2} \tag{9}$$

Substituting Equations (7) through (9) into Equation (5) with the aid of Equation (4), one obtains

$$\frac{D^2\phi}{Dx^2} = \frac{1}{x^2}\left\{\frac{1}{\bar{\gamma}M^2P}\left(x^2\frac{D\phi}{Dx}\frac{\partial P}{\partial x} - \frac{\partial P}{\partial \phi}\right)\left[1 + \left(x\frac{D\phi}{Dx}\right)^2\right] - x\frac{D\phi}{Dx}\left[2 + \left(x\frac{D\phi}{Dx}\right)^2\right]\right\} \tag{10}$$

Although Equation (10) is appropriate for calculating the streamline location in terms of x and ϕ, it must be recast into a first-order equation for the following reasons:

1. It is a second-order equation to which the solution $\phi = \phi(x)$ along a streamline is obtained through double integration and the accumulated error in numerical integration will be greater than a first-order one.
2 Further differentiation will be involved in deriving the equations for equivalent radius, and it is convenient to use a first-order equation as the basic differential equation.

Accordingly, a new variable θ, the angle between the tangent of local streamline and x-axis, is given by the relation

$$\theta = \tan^{-1}\left(\frac{v}{u}\right) = \tan^{-1}\left(x\,\frac{D\phi}{Dx}\right) \tag{11}$$

Differentiating Equation (11) with respect to x along a streamline and rearranging, one finds that

$$\frac{D^2\phi}{Dx^2} = \frac{1}{x\cos^2\theta}\,\frac{D\theta}{Dx} - \frac{\tan\theta}{x^2} \tag{12}$$

The differential equation for determining the local direction of a streamline is found by equating Equations (10) and (12) and rearranging:

$$\frac{D\theta}{Dx} = \frac{1}{\gamma M^2 P}\left(\tan\theta\,\frac{\partial P}{\partial x} - \frac{1}{x}\,\frac{\partial P}{\partial \phi}\right) - \frac{\tan\theta}{x} \tag{13}$$

Also from Equation (11),

$$\frac{D\phi}{Dx} = \frac{\tan\theta}{x} \tag{14}$$

Equations (13) and (14) are valid along a streamline and so they can be integrated simultaneously to determine the geometry of a chosen streamline $\theta = \theta(x)$ and $\phi = \phi(x)$. However, the derivatives $D\phi/Dx$ and $D\theta/Dx$ become infinite when θ = 90 degrees. In connection with these infinite derivatives, it is helpful to rewrite Equations (13) and (14) by using S (the distance measured along a streamline) as an independent variable instead of x. Accordingly, along a streamline:

$$\frac{D\phi}{DS} = \frac{D\phi}{Dx}\,\frac{Dx}{DS}$$

Since

$$\frac{Dx}{DS} = \cos\,\theta$$

therefore

$$\frac{D\phi}{DS} = \frac{\sin\theta}{x} \tag{15}$$

and Equation (13) becomes

$$\frac{D\theta}{DS} = \frac{1}{\bar{\gamma}M^2P}\left(\sin\theta\,\frac{\partial P}{\partial x} - \frac{\cos\theta}{x}\,\frac{\partial P}{\partial\phi}\right) - \frac{\sin\theta}{x} \tag{16}$$

As a result of the change of the independent variables, x now becomes a dependent variable. Hence, x can be calculated simultaneously with θ and ϕ from the relation:

$$\frac{Dx}{DS} = \cos\,\theta \tag{17}$$

Equations (15), (16), and (17) constitute a set of simultaneous, first-order, ordinary differential equations for determining the geometry of a chosen streamline from a known pressure distribution. The integration gives different streamlines for different initial conditions.

EQUATIONS FOR EQUIVALENT RADIUS

The previous section provided the necessary equations for determining the streamline geometry. However, a very important and essential variable, namely, the equivalent radius, has to be determined in order to calculate the three-dimensional heat transfer using "axisymmetric analogue." It is the scale factor of the coordinate normal to the streamline in the streamwise system. Physically, it is a measure of the spreading distance between two adjacent streamlines.

The desired new system (ξ,β) is also orthogonal, as mentioned previously. The two systems are related analytically. Thus,

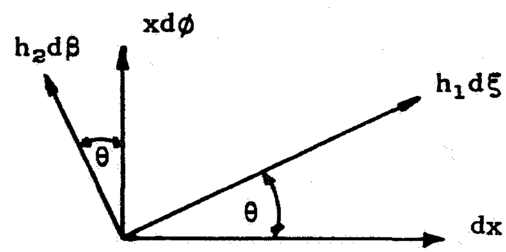
$$x = x(\xi,\beta), \qquad \phi = \phi(\xi,\beta) \tag{18}$$

Their differentials are

$$dx = \frac{\partial x}{\partial \xi} d\xi + \frac{\partial x}{\partial \beta} d\beta \tag{19a}$$

$$d\phi = \frac{\partial \phi}{\partial \xi} d\xi + \frac{\partial \phi}{\partial \beta} d\beta \tag{19b}$$

Also with the aid of the following sketch,



one finds that:

$$dx = h_1 \cos\theta \, d\xi - h_2 \sin\theta \, d\beta \tag{20a}$$

$$d\phi = \frac{h_1 \sin\theta}{x} d\xi + \frac{h_2 \cos\theta}{x} d\beta \tag{20b}$$

Equating the coefficients in Equations (19) and (20), it follows that

$$\frac{\partial x}{\partial \xi} = h_1 \cos\theta \tag{21a}$$

$$\frac{\partial x}{\partial \beta} = -h_2 \sin\theta \tag{21b}$$

$$\frac{\partial \phi}{\partial \xi} = \frac{h_1 \sin\theta}{x} \tag{21c}$$

$$\frac{\partial \phi}{\partial \beta} = \frac{h_2 \cos\theta}{x} \tag{21d}$$

Since $x = x(\xi,\beta)$ and $\phi\ (\xi,\beta)$, then

$$\frac{\partial^2 x}{\partial\beta\,\partial\xi} = \frac{\partial^2 x}{\partial\xi\,\partial\beta} \tag{22a}$$

$$\frac{\partial^2 \phi}{\partial\beta\,\partial\xi} = \frac{\partial^2 \phi}{\partial\xi\,\partial\beta} \tag{22b}$$

For Equation (22a)

$$\frac{\partial}{\partial\beta}\left(\frac{\partial x}{\partial\xi}\right) = \cos\theta\,\frac{\partial h_1}{\partial\beta} - h_1\sin\theta\,\frac{\partial\theta}{\partial\beta}$$

$$\frac{\partial}{\partial\xi}\left(\frac{\partial x}{\partial\beta}\right) = -\sin\theta\,\frac{\partial h_2}{\partial\xi} - h_2\cos\theta\,\frac{\partial\theta}{\partial\xi}$$

Equating the right-hand sides of the above two equations gives

$$\frac{1}{h_1}\,\frac{\partial h_2}{\partial\xi} = \frac{\partial\theta}{\partial\beta} - \frac{h_2\cot\theta}{h_1}\,\frac{\partial\theta}{\partial\xi} - \frac{\cot\theta}{h_1}\,\frac{\partial h_1}{\partial\beta} \tag{23}$$

For Equation (22b):

$$\frac{\partial}{\partial\beta}\left(\frac{\partial\phi}{\partial\xi}\right) = \frac{\sin\theta}{x}\,\frac{\partial h_1}{\partial\beta} + \frac{h_1\cos\theta}{x}\,\frac{\partial\theta}{\partial\beta} + \frac{h_1h_2\sin^2\theta}{x^2}$$

and

$$\frac{\partial}{\partial\xi}\left(\frac{\partial\phi}{\partial\beta}\right) = \frac{\cos\theta}{x}\,\frac{\partial h_2}{\partial\xi} - \frac{h_2\sin\theta}{x}\,\frac{\partial\theta}{\partial\xi} - \frac{h_1h_2\cos^2\theta}{x^2}$$

Equating the right-hand sides of the above two equations gives

$$\frac{\partial h_1}{\partial\beta} = \cot^2\theta\,\frac{\partial h_2}{\partial\xi} - h_1\,\frac{\partial\theta}{\partial\beta} - h_2\,\frac{\partial\theta}{\partial\xi} - \frac{h_1h_2}{x\sin\theta}$$

Substituting the above equation into Equation (23) and simplifying, one obtains along a streamline

$$\frac{Dh_2}{DS} = \frac{1}{h_1}\,\frac{\partial h_2}{\partial\xi} = \frac{\partial\theta}{\partial\beta} + \frac{h_2\cos\theta}{x} \tag{24}$$

The term $(\partial\theta/\partial\beta)$ in Equation (24) is obtained by differentiating Equation (16) with respect to β letting

$$\frac{\partial^2\theta}{\partial\beta\;\partial\xi} = \frac{\partial^2\theta}{\partial\xi\;\partial\beta} \tag{25}$$

The result is as follows:

$$\frac{D}{DS}\left(\frac{\partial\theta}{\partial\beta}\right) = \frac{1}{h_1}\frac{\partial}{\partial\xi}\left(\frac{\partial\theta}{\partial\beta}\right) = \frac{\partial}{\partial\beta}\left(\frac{D\theta}{DS}\right) + \frac{1}{h_1}\frac{D\theta}{DS}\frac{\partial h_1}{\partial\beta}$$

or

$$\frac{D}{DS}\left(\frac{\partial\theta}{\partial\beta}\right) = \left[\frac{1}{\overline{\gamma}M^2P}\left(\cos\theta\,\frac{\partial P}{\partial x} + \frac{\sin\theta}{x}\,\frac{\partial P}{\partial\phi}\right) - \frac{\cos\theta}{x}\right]\frac{\partial\theta}{\partial\beta}$$

$$- h_2\left\{\frac{D\theta}{DS}\left(\frac{D\theta}{DS} + \frac{\sin\theta}{x}\right) + \frac{\sin^2\theta}{x^2}\right.$$

$$+ \frac{1}{\overline{\gamma}M^2P}\left[\sin^2\theta\,\frac{\partial^2P}{\partial x^2} - \sin\theta\cos\theta\left(\frac{2}{x}\,\frac{\partial^2P}{\partial\phi\partial x} - \frac{1}{x^2}\,\frac{\partial P}{\partial\phi}\right)\right.$$

$$\left.\left. + \frac{\cos^2\theta}{x^2}\,\frac{\partial^2P}{\partial\phi^2}\right] + \frac{2-M^2}{\overline{\gamma}M^2P}\left(\sin\theta\,\frac{\partial P}{\partial x} - \frac{\cos\theta}{x}\,\frac{\partial P}{\partial\phi}\right)^2\right\} \tag{26}$$

EQUATIONS FOR CYLINDRICAL LEADING EDGE

It is necessary to employ other body coordinates (x,ϕ) for the cylindrical leading edge (see Figure 3). Following the same procedure, the differential equations for the streamline geometry are:

$$\frac{Dx}{DS} = \cos\theta \tag{27}$$

$$\frac{D\phi}{DS} = \frac{\sin\theta}{r} \tag{28}$$

$$\frac{D\theta}{DS} = \frac{1}{\overline{\gamma}M^2P}\left(\sin\theta\,\frac{\partial P}{\partial x} - \frac{\cos\theta}{r}\,\frac{\partial P}{\partial\phi}\right) \tag{29}$$

Those for the equivalent radius are:

$$\frac{Dh_2}{DS} = \frac{\partial\theta}{\partial\beta} \tag{30}$$

$$\frac{D}{DS}\left(\frac{\partial\theta}{\partial\beta}\right) = \frac{1}{\bar{\gamma}M^2P}\left\{\left(\cos\theta\,\frac{\partial P}{\partial x} + \frac{\sin\theta}{r}\,\frac{\partial P}{\partial\phi}\right)\frac{\partial\theta}{\partial\beta}\right.$$

$$- h_2\left[\sin^2\theta\,\frac{\partial^2 P}{\partial x^2} - \frac{\sin 2\theta}{r}\,\frac{\partial^2 P}{\partial\phi\,\partial x}\right.$$

$$\left.\left. + \frac{\cos^2\theta}{r^2}\,\frac{\partial^2 P}{\partial\phi^2}\right] + \frac{2 - M^2}{\bar{\gamma}M^2P}\left(\sin\theta\,\frac{\partial P}{\partial x} - \frac{\cos\theta}{r}\,\frac{\partial P}{\partial\phi}\right)^2\right\} - h_2\left(\frac{D\theta}{DS}\right)^2 \tag{31}$$

ESTIMATION OF SURFACE PRESSURE DISTRIBUTION

As indicated in the previous analysis, the surface pressure distribution must be known in order to calculate the streamline geometry and equivalent radius. Two pressure estimation techniques are used for the flat delta wing surface. The first is the Newtonian-conical flow theory given by Hida[10]

$$\frac{P}{P_o} = C_1 + C_2\tan^2\phi \tag{32}$$

where C_1 and C_2 are functions of M_∞, α, and the span b. Determination of C_1 and C_2 involves rather lengthy manipulation and thus it will not be repeated here. At a small angle of attack where C_2 is negative, streamlines converge to the center. When the angle of attack is large, C_2 becomes positive, which in turn, forms a diverging streamline pattern. This phenomenon agrees qualitatively with experimental results.

The second pressure estimation technique is that of Creager[11] as modified by Buck and McLaughlin.[12]

$$P = \left(P_{\text{surf. inclin.}} + P_{\text{shock curv.}}\right)\left(1 + E_{\text{viscous}}\right) \tag{33}$$

where

$$P_{\text{surf inclin}} = \begin{cases} \text{(a) Pressure after oblique shock for small angle of attack (shock attached)} \\ \text{(b) Pressure by modified Newtonian for large angle of attack (shock detached)} \end{cases}$$

$P_{\text{shock curv.}}$ = Pressure from blast wave theory

E_{viscous} = Correction due to boundary layer displacement

The details are found in Reference 12. However, this expression does not give diverging flow patterns at any angle of attack.

As suggested in Reference 12, an empirical expression for pressure distribution (from Gregorek[13]) was used for the cylindrical leading edge. Thus:

$$\frac{P}{P_o} = 0.32 + 0.455 \cos \phi + 0.195 \cos 2\phi + 0.035 \cos 3\phi - 0.005 \cos 4\phi \tag{34}$$

This expression is good for hypersonic flows.

A flat delta wing with cylindrical leading edges normally has a blunt nose. The nose may be a body of revolution, a blunted cone, or simply a shperical cap. The aerodynamic stagnation point lies on the nose from which streamlines emanate. Therefore, if the flow pattern is a converging one (see Figure 1, Case I), the streamline geometry and the equivalent radius must be calculated first for the nose region, then for the leading edge, and finally for the delta wing surface. The sequence for a diverging flow (Case II) is nose first, then wing surface, and finally leading edge. The simplest nose configuration is a spherical cap. An exact geometric solution of the streamline geometry and equivalent radius can be obtained merely through transformation of coordinates; see the Appendix. The method given in Reference 9 is appropriate for other nose configurations such as a blunted body of revolution.

RESULTS AND DISCUSSION

Figures 4-6 show calculated streamline geometries over the windward side of flat delta wings with a cylindrical leading edge under hypersonic flow conditions at different angles of attack. The flow conditions were chosen to permit comparison with available theoretical and experimental data.

Figure 4 gives results for the case of a 75 degree swept delta wing at $M_\infty = 9.6$ and $\alpha = 30$ and 60 degrees. The Newtonian-conical pressure was used for the present method. At $\alpha = 30$ degrees, it yielded a converging streamline pattern. As the angle of attack increased, $\alpha = 60$ degrees, the flow shifted toward a diverging pattern. Figure 4 also indicates results for the perturbation method of Polak and Li[14] and for experimental data taken by Bertram and Henderson.[15] Note that the present method shows better agreement with the experimental data than does the perturbation method.

The extent to which results depend on the pressure distributions utilized in calculations is demonstrated by Figure 5 which shows streamline patterns over a 60 degree swept delta wing at $M_\infty = 8.0$ and $\alpha = 30$ degrees obtained by two different techniques for estimating pressure: (1) the Newtonian-conical pressure (Equation (32)) and (2) the hybrid pressure (Equation (33)). Quite different streamline patterns were obtained even under the same flow condition. The difference became larger near the axis of symmetry of the wing. Physically it is impossible for any streamline to cross the axis of symmetry. The streamlines given by the Newtonian-conical pressure appear to cross that line because a finite pressure gradient in the ϕ direction still exists at the axis of symmetry.

(This particular drawback of the Newtonian-conical theory ought to be eliminated.)

The streamlines shown in Figure 5 come from the cylindrical leading edge of the wing. Figure 6 gives results for this portion of the surface as obtained from the present method using the empirical pressure distribution (Equation (34)). The flow conditions are the same as in Figure 5. As a consequence of the angle of attack, the $\phi = 30$ degree line on the cylinder becomes the margin that divides the windward and leeward sides. Streamlines lying above that dividing line will run into the windward side of the wing; all others will go to the leeward side. The results on this cylindrical leading edge provide the starting conditions for integration along the wing surface.

The equivalent radius can also be easily calculated with the same pressure distribution as used for the streamline geometry. The equivalent radius and its associated application to hypersonic heat transfer has been computed by Sacks*.

CONCLUSIONS

The following conclusions are drawn from the present study:

1. Calculations of streamline geometry depend heavily on the pressure distribution used. The more realistic the pressure distribution, the more exact the method.

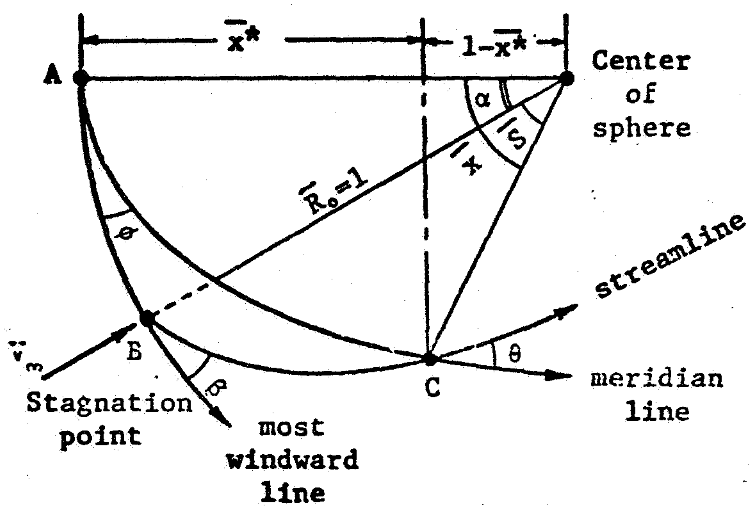
2. The streamline geometry obtained from the present method using the Newtonian-conical pressure for a flat delta wing at hypersonic speeds correlates better with experimental data than does that obtained by the perturbation method.

*Reported informally in NSRDC Tech Note AL-203

APPENDIX

GEOMETRIC SOLUTION FOR A SPHERICAL CAP

With the aid of the following sketch,



Reference 9 showed that:

$$\overline{x} = \text{Arccos}(\cos\alpha \cos \overline{S} - \sin\alpha \sin \overline{S} \cos\beta) \tag{A-1}$$

$$\overline{x}* = 1 - \cos \overline{x} \tag{A-2}$$

$$\phi = \text{Arcsin}\left(\frac{\sin \overline{S} \sin\beta}{\sin \overline{x}}\right) \tag{A-3}$$

$$\theta = \text{Arcsin}\left(\frac{\sin\alpha \sin\beta}{\sin \overline{x}}\right) \tag{A-4}$$

and

$$\overline{h}_2 = \sin \overline{S} \tag{A-5}$$

where

$$\overline{x} = \frac{x}{R_o}, \quad \overline{x}* = \frac{x*}{R_o}, \quad \overline{S} = \frac{S}{R_o} \text{ and } \overline{h}_2 = \frac{h_2}{R_o}$$

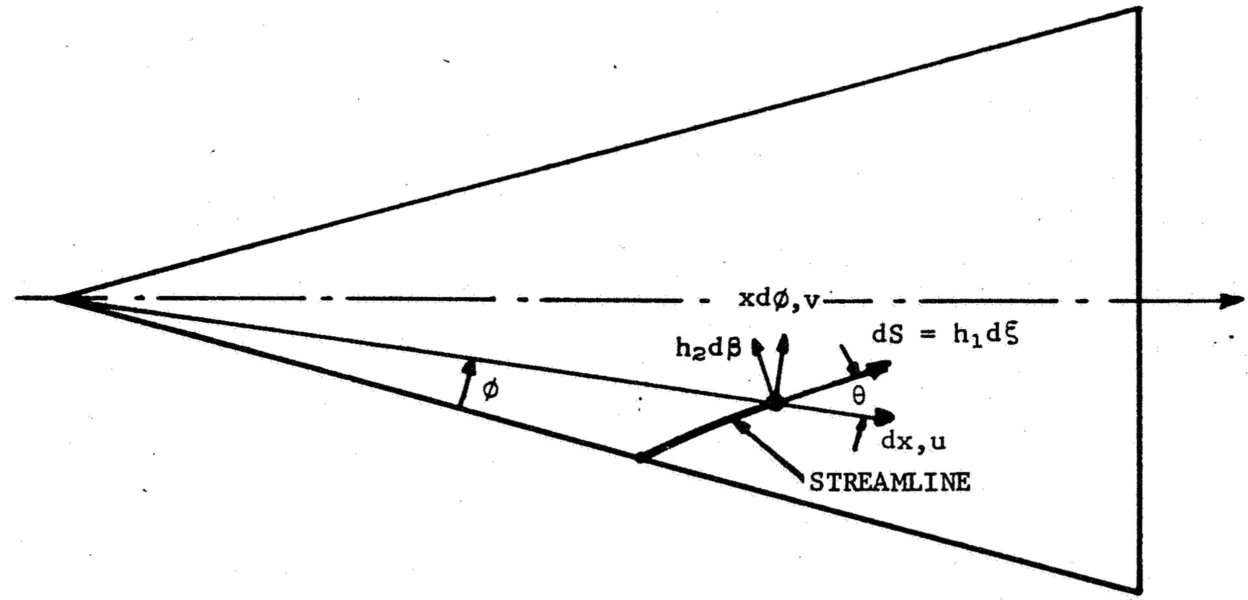


Figure 1a - Case I - Converging Flow Pattern

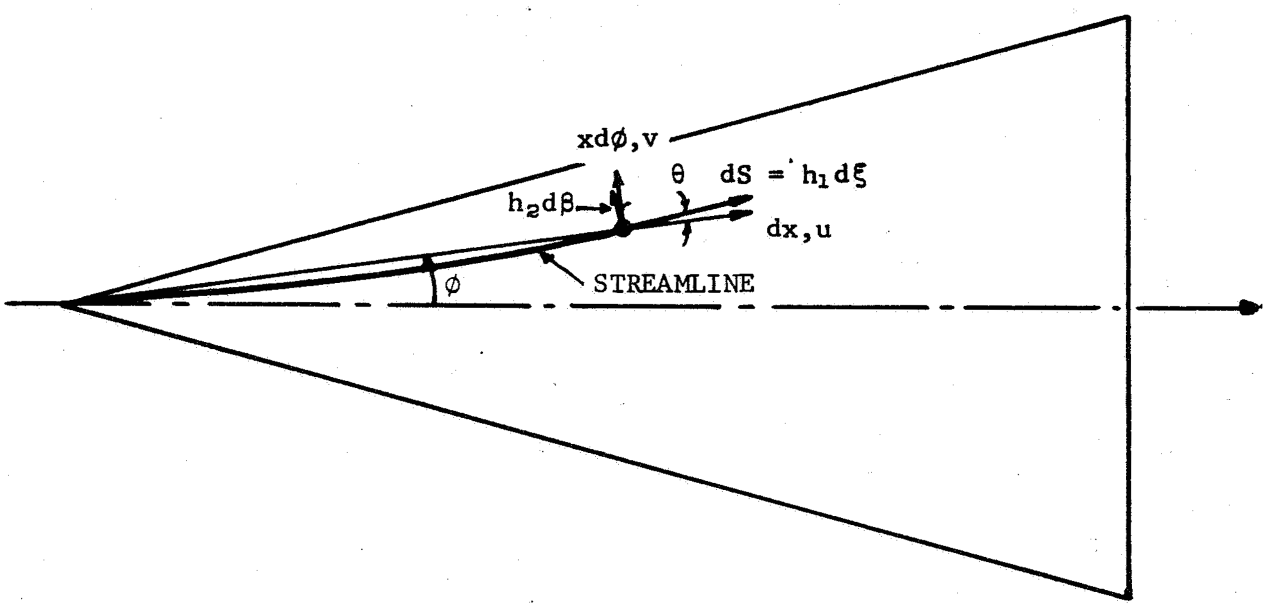


Figure 1b - Case II - Diverging Flow Pattern

Figure 1 - Coordinate Systems for Flat Delta Wing

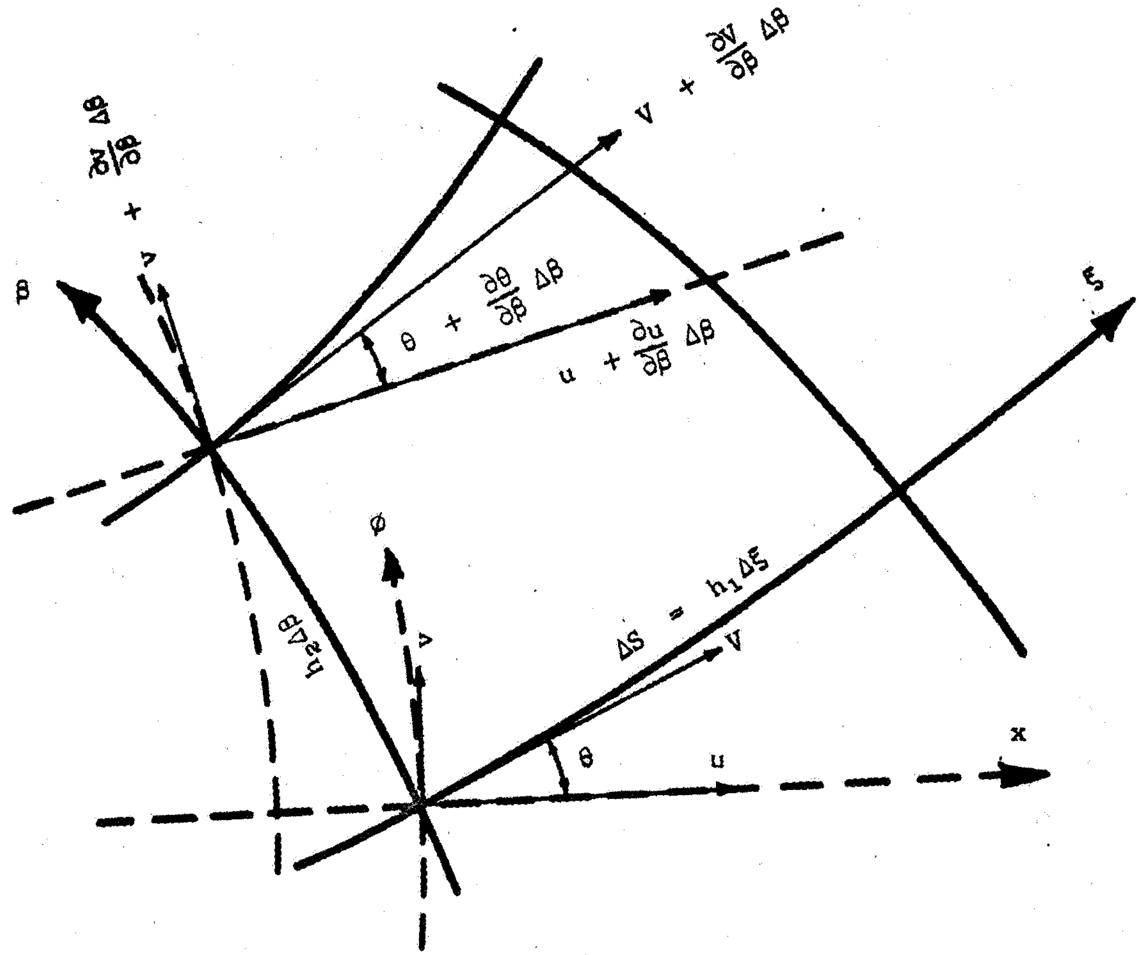


Figure 2 - Streamwise (ξ,β) and Orthogonal (x,ϕ) Coordinates on the Body Surface

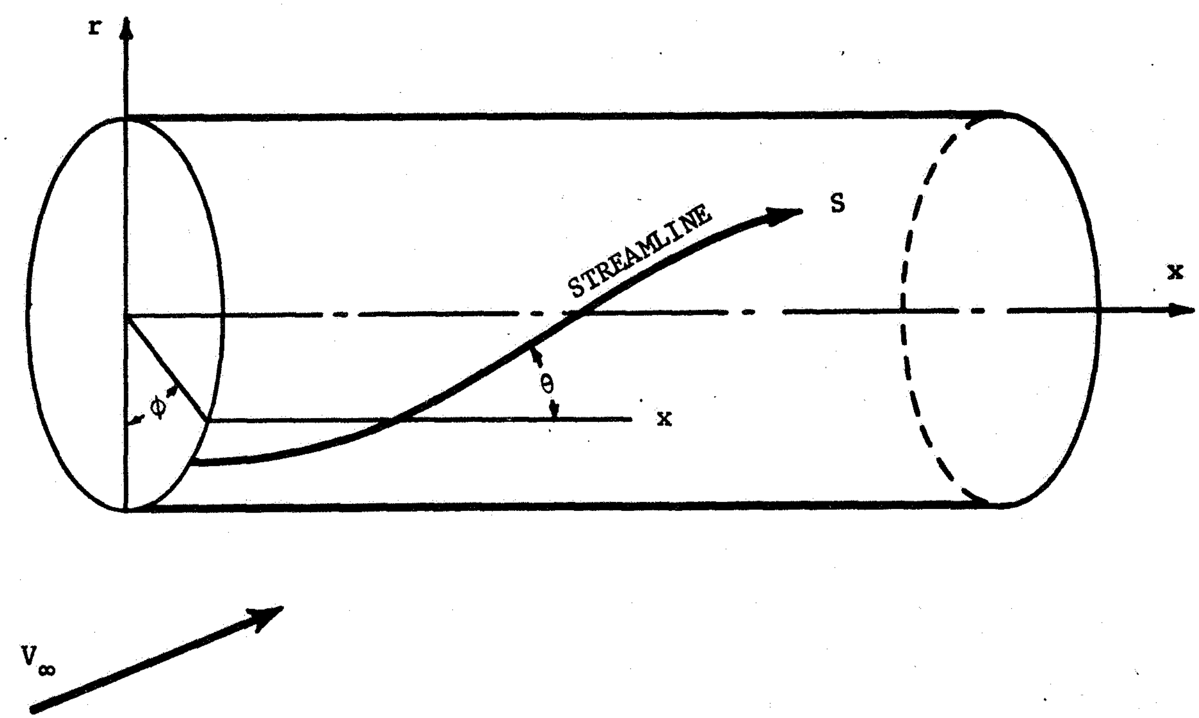


Figure 3 - Coordinates for a Yawed Cylinder

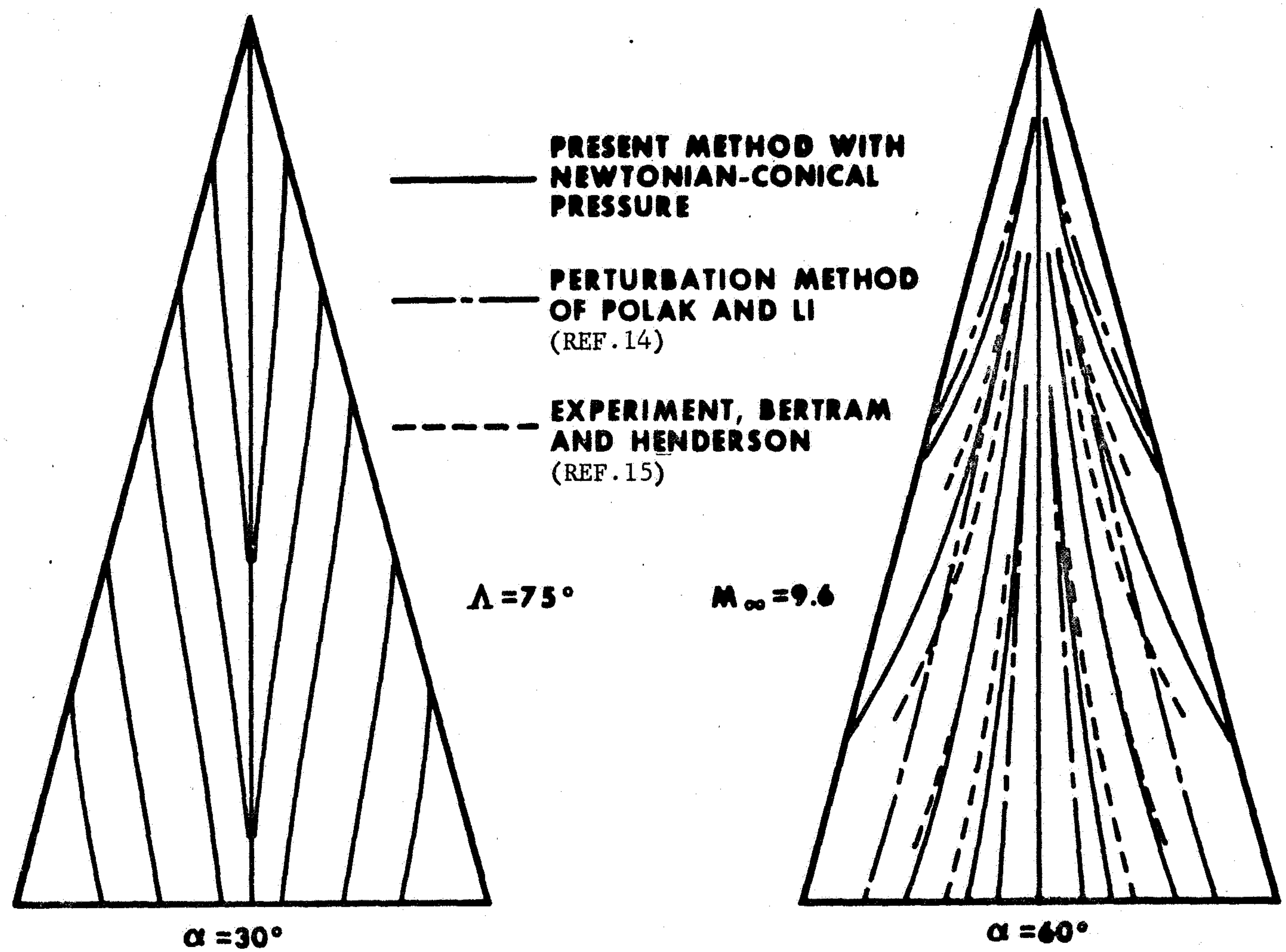


Figure 4 - Streamline Pattern for a Flat Delta Wing at $M_\infty = 9.6$

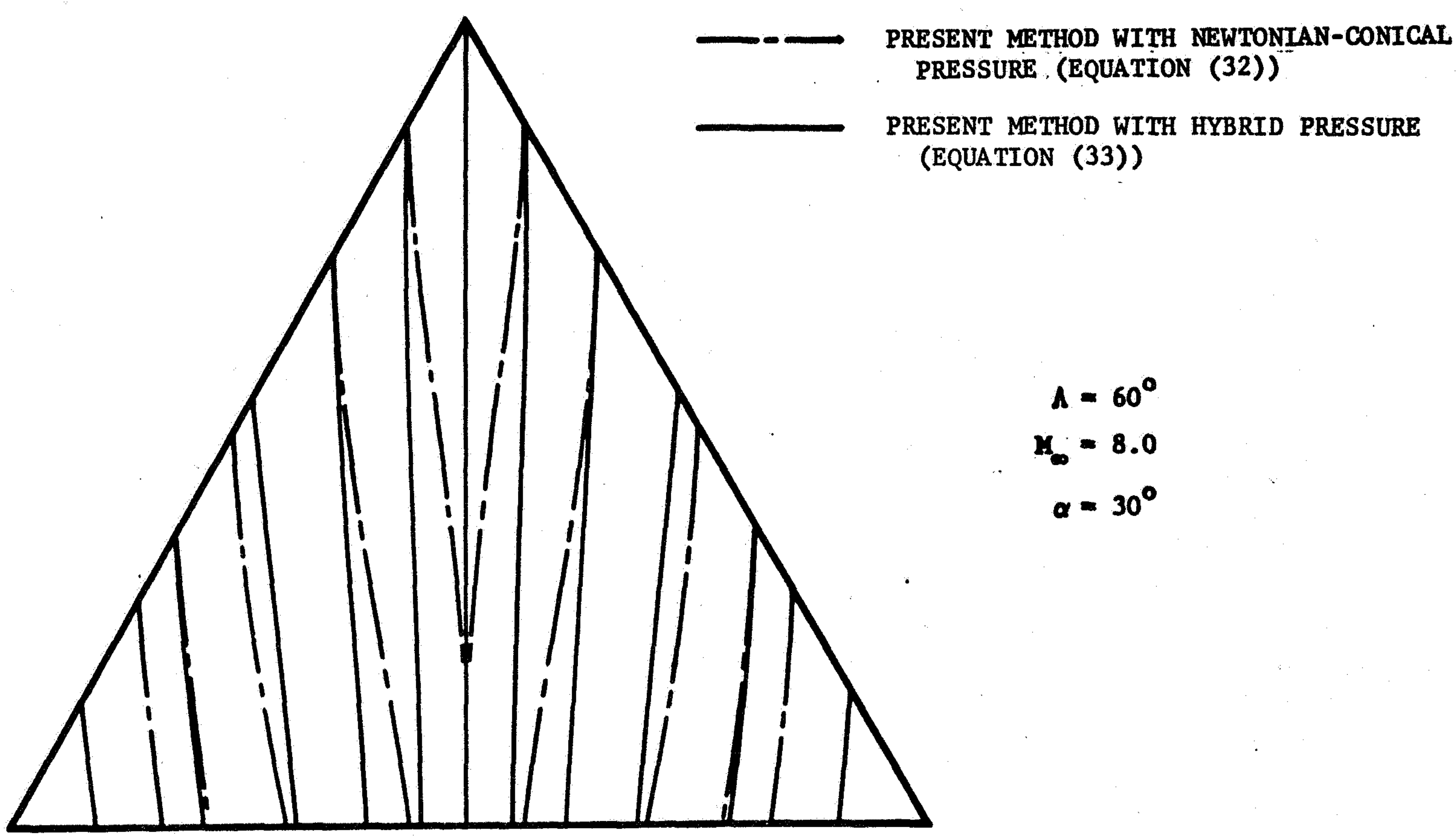


Figure 5 - Streamline Pattern for a Flat Delta Wing at α = 30 Degrees and M_{∞} = 8.0

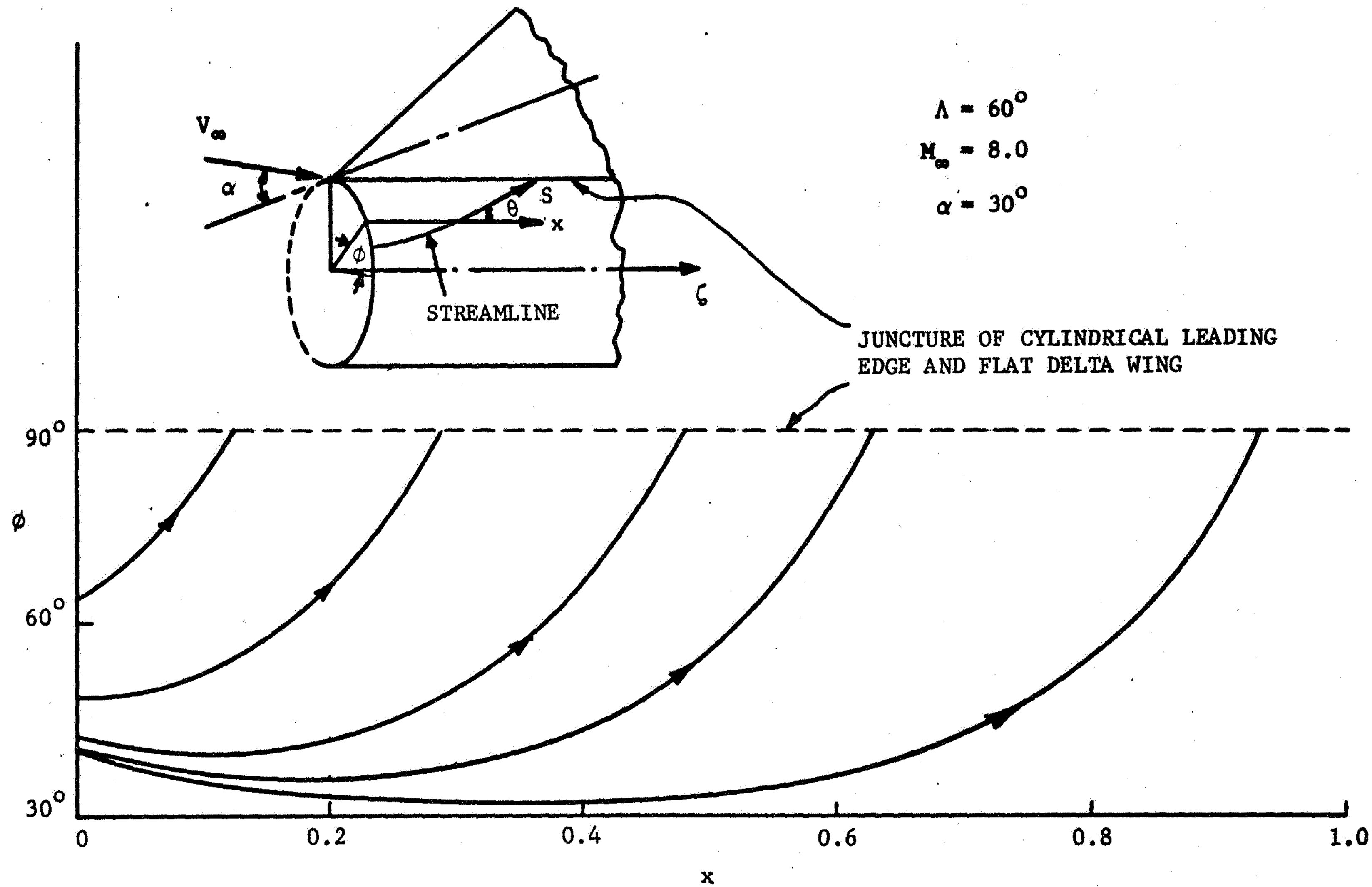


Figure 6 - Streamline Pattern for the Cylindrical Leading Edge of a Flat Delta Wing at α = 30 Degrees and M_∞ = 8.0

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13. ABSTRACT

An exact method has been developed for determining the streamline geometry and equivalent radius (the scale factor for the normal coordinate in a streamwise coordinate system) over a flat delta wing with cylindrical leading edge at angles of attack. This method requires a knowledge of the surface inviscid pressure distribution, either theoretical or experimental. With the aid of the present method, three-dimensional hypersonic heat transfer can be simply calculated through the axisymmetric analogue.

Results are presented for flat delta wings traveling at hypersonic speeds and at various angles of attack. Calculated results indicate that the streamline geometry depends heavily on pressure distribution used in the calculation. The present method gives better correlation with experimental data than does the perturbation method.

14. KEY WORDS	LINK A ROLE	LINK A WT	LINK B ROLE	LINK B WT	LINK C ROLE	LINK C WT
Streamline geometry						
Equivalent radius						
Delta wing						
Angle of attack						